**BASIC EQUATIONS FOR HEAT TRANSFER**

Conduction:

\[
q = k \frac{dT}{dx}
\]

(1)

Convection:

\[
q = \bar{h}_{av} A_{s} (T_1 - T_f)
\]

(2)

Radiation

\[
q = \varepsilon \sigma (T^4 - T_{w}^4)
\]

(3)

**HEAT TRANSFER THROUGH SHELL-AND-TUBE HEAT EXCHANGERS**

Heat duty for exchanger transferring sensible heat:

\[
q = \dot{m} C_p \cdot \Delta T
\]

(4)

For use in heat-exchanger calculations, Equation (2) above is often written as follows:

\[
q = UA \Delta T
\]

(5)

where \( U \) can be calculated from the following relationship:

\[
\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + \frac{1}{h_w} + \frac{1}{h_{avg}}
\]

(6)

Various equations are available (see the references) for calculating \( h_i \) and \( h_o \), depending on such factors as the Reynolds numbers for the flowing fluids and whether the fluids undergo sensible heat transfer or, instead, vaporization or condensation. For instance, for sensible heat transfer with fluids under forced convection in fully turbulent flow inside tubes with sharp-edged entrances, the following, well established relationship involving the Nusselt, Reynolds and Prandtl numbers holds:

\[
h_D = \frac{0.023 \cdot \bar{h}_{avg} \cdot D_m}{\mu} \cdot \frac{\rho^0.8 \cdot k^0.4}{\mu^0.4}
\]

(7)

With the assumption that the \( \mu/\mu_w \) term can be ignored, the immediately above equation has been rearranged as follows [1, 2] to facilitate assessing the effects of fluid (and system) properties upon \( h \) (assuming sensible heat transfer, full turbulence, fluid inside tubes):

\[
h = \frac{0.23 \cdot \bar{h}_{avg} \cdot D_m^{0.8} \cdot k^{0.4}}{\mu^0.4}
\]

(8)

For sensible heat transfer with fluids under forced convection flowing across tube banks (thus, outside the tubes), the following relationship has been published [3]

\[
h = \frac{0.8 \cdot \bar{h}_{avg} \cdot D_m^{0.8} \cdot k^{0.4}}{\mu^0.4}
\]

(9)

In this equation, the values of \( \alpha \) and \( m \) are to be as follows:

<table>
<thead>
<tr>
<th>Tube pattern</th>
<th>Reynolds number</th>
<th>( m )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Staggered</td>
<td>above 200,000</td>
<td>0.300</td>
<td>0.166</td>
</tr>
<tr>
<td>Staggered</td>
<td>300 to 200,000</td>
<td>0.365</td>
<td>0.237</td>
</tr>
<tr>
<td>Staggered</td>
<td>less than 300</td>
<td>0.640</td>
<td>1.309</td>
</tr>
<tr>
<td>Inline</td>
<td>above 200,000</td>
<td>0.300</td>
<td>0.124</td>
</tr>
<tr>
<td>Inline</td>
<td>300 to 200,000</td>
<td>0.349</td>
<td>0.211</td>
</tr>
<tr>
<td>Inline</td>
<td>less than 300</td>
<td>0.569</td>
<td>0.742</td>
</tr>
</tbody>
</table>

For an excellent discussion of the Equation (6) fouling factor, \( h_o \), see Reference [4].

The appropriate \( \Delta T \) depends on the configuration of the heat exchanger (see references). For example, for a simple counter-current flow exchanger, the appropriate temperature (referred to as the log mean temperature difference, \( \Delta T_{LM} \), is found as follows:

\[
\Delta T_{LM} = \frac{(T_b - T_o) - (T_o - T_c)}{\ln(T_b - T_o) / (T_o - T_c)}
\]

(10)

Energy balance for a heat exchanger

If any heat exchange with the ambient air is neglected, the following relationship is valid:

\[
\dot{m}_h (H_{ia} - H_{ib}) = \dot{m}_c (H_{cb} - H_{co}) = q
\]

(11)

**BATCH HEATING**

For heating a batch of fluid from temperature \( T_1 \) to \( T_2 \), by means of an internal coil of area \( A \) and an isothermal heating medium at temperature \( T \), the following relationship holds:

\[
\ln \left( \frac{T - T_1}{T_2 - T} \right) = \frac{(UA)}{c \cdot M} \cdot t
\]

(12)

**STEADY-STATE HEAT FLOW BY CONDUCTION**

For conduction through a homogeneous plane wall of thickness \( x \) and constant (or average) thermal conductivity \( k \),

\[
q = k \frac{\Delta T}{x}
\]

(13)

where \( \Delta T \) is the temperature difference through the wall.

For conduction through a three-layered plane wall (for example, a wall with thermal insulation on each side), having layers of thicknesses \( x_1, x_2 \) and \( x_3 \) and thermal conductivities \( k_1, k_2 \) and \( k_3 \),

\[
q = \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{x_3}{k_3}
\]

(14)

where \( \Delta T \) is the overall temperature difference across all three layers.

For conduction through the wall of a cylinder of length \( L \), whose inner and outer radii are \( r_{inner} \) and \( r_{outer} \), with inner and outer walls at temperatures \( T_{inner} \) and \( T_{outer} \),

\[
q = k(2\pi L)[T_{inner} - T_{outer}]
\]

(15)

\[
\ln (r_{outer} / r_{inner})
\]

**References**


**NOMENCLATURE:**

- \( A \) cross-sectional area perpendicular to the flow of heat
- \( a \) parameter in convection-coefficient equation
- \( A_s \) surface area
- \( c, c_p \) specific heat; specific heat at constant pressure
- \( C_p, \text{avg} \) specific heat at average fluid temperature
- \( D_i \) inner diameter of heat-exchanger tube
- \( D_o \) outer diameter of heat-exchanger tube
- \( H_{ia}, H_{ib} \) enthalpy per unit mass of entering cold and warm fluid, respectively
- \( h_{avg} \) average convection coefficient
- \( h_i \) convection coefficient for inner tube wall
- \( h_o \) convection coefficient for outer tube wall
- \( h_f \) fouling heat-transfer coefficient
- \( h_w \) coefficient of heat-transfer radially through tube wall; a function of tube thickness and thermal conductivity
- \( k \) thermal conductivity
- \( L \) length
- \( m \) weight of batch
- \( m_f \) mass flow rate of fluid
- \( m_c \) mass flow rate of cold fluid
- \( m_h \) mass flow rate of hot fluid
- \( q \) rate of heat flow
- \( T \) temperature (for radiation calculations, use absolute temperature)
- \( T_e \) in a heat exchanger, the exit temperature for the stream being cooled
- \( T_r \) temperature of fluid
- \( T_h \) in a heat exchanger, the inlet temperature for the stream being cooled
- \( T_{inlet} \) inlet temperature
- \( T_{outlet} \) outlet temperature
- \( T_s \) temperature of surface (for radiation calculations, use absolute temperature)
- \( T_{sur} \) temperature of surroundings (for radiation calculations, use absolute temperature)
- \( x \) in a heat exchanger the inlet temperature for the stream being heated
- \( x_{inner} \) in a heat exchanger, the outlet temperature for the stream being heated
- \( \Delta T \) temperature difference
- \( \Delta T_{LM} \) temperature gradient during conductive heat flow
- \( U \) overall heat transfer coefficient
- \( x \) distance the heat flows during conduction
- \( \varepsilon \) emissivity
- \( \mu \) viscosity; viscosity at bulk fluid temperature
- \( \mu_w \) viscosity at wall temperature
- \( \alpha \) Stefan-Boltzmann constant
- \( \theta \) time required for batch heating